

***PJ*-SLIGHTLY δ - β -CONTINUOUS FUNCTIONS**

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ABSTRACT

The aim of this paper is to introduce and study a new classes of functions is called *pj*-slightly δ - β -continuous by using *ij*- δ - β -open sets.

KEYWORDS: Bitopological Spaces, *Ij*- $\mathcal{A}$ -*B*-Open Sets, *Pj*-Slightly $\mathcal{A}$ -*B*-Continuous Functions, *Ij*- $\mathcal{A}$ - *B* -Irresolute

1. INTRODUCTION

The concept of δ - β -continuous functions with the aid of δ - β -open sets have been developed by Hatir and Noiri [2] . Rajesh et al. [13] have introduced the notion of quasi δ - β - continuous functions. Jafari and Rajesh have introduced the concept of almost δ - β -continuous functions [4] and faintly δ - β -continuous functions [5]. After their work, have been circulated the concept *ij*- δ - β -open and *ij*- δ - β -continuous function in bitopological space by Albeladi and Alblowi [1]. Also Alblowi and Albeladi [1] have introduced the notion of *pj*- quasi , *pj*- almost and *pj*-faintly δ - β - continuous functions. The aim of this paper is to introduce and study a new classes of functions is called *pj*-slightly δ - β -continuous.

2. PRELIMINARIES

Let *A* be a subset of a bitopological space (X, τ^1, τ^2) . We denote the *i*-closure of *A* and the *i*-interior of *A* by *i*- *Cl*(*A*) and *i*- *Int*(*A*), respectively with τ^i for $i = 1, 2$. A subset *A* of a bitopological space (X, τ^1, τ^2) is said to be *ij*-regular open [7] if $A = i\text{-int}(j\text{-Cl}(A))$. A set *A* is said to be *pj*-regular open (resp. *jp*-regular open) if $A = j\text{-int}(p\text{-Cl}(A))$ (resp. $A = P\text{-int}(j\text{-cl}(A))$) [9]. The complement of a *ij*-regular open (resp. *pj*-regular open, *jp*-regular open) set is called *ij*-regular closed (resp. *pj*-regular closed, *jp*-regular closed). A subset *A* of space (X, τ^1, τ^2) is called *ij*-semiopen if $A \subset j\text{-Cl}(i\text{-int}(A))$ [2]. A point *x* of *X* is said to be *ij*- δ -cluster point of *A* if $A \cap U \neq \emptyset$ for every *ij*-regular open set *U* containing *x* [7]. The set of all *ij*- δ - cluster points of *A* is called *ij*- δ -closure of *A* and is denoted by $ij\text{-Cl}^\delta(A)$ [7]. A subset *A* of *X* is said to be *ij*- δ -closed if *ij*- δ -cluster points of *A* $\subset A$ [7]. The complement of *ij*- δ -closed set is *ij*- δ -open. So a set is *ij*- δ -open if it is expressible as a union of *ij*-regular open sets[7]. A point $x \in X$ is said to be *pj*- θ -cluster point of *A* if $A \cap j\text{-Cl}(V) \neq \emptyset$ for every *p*-open set *V* containing *x*. The set of all *pj*- θ -cluster points of *A* is called the *pj*- θ -closure of *A* and is denoted by $pj\text{-Cl}^\theta(A)$. If $A = pj\text{-Cl}^\theta(A)$, then *A* is said to be *pj*- θ -closed . The complement of *pj*- θ -closed set is said to be *pj*- θ -open. The union of all *pj*- θ -open sets contained in a subset *A* is called the *pj*- θ -interior of *A* and is denoted by $pj\text{-Int}^\theta(A)$. A subset *A* of a bitopological space (X, τ^1, τ^2) is said to be *ij*- δ - β -open if $S \subset j\text{-Cl}(i\text{-Int}(ij\text{-Cl}^\delta(S)))$. The complement of an *ij*- δ - β -open set is called *ij*- δ - β -closed. The intersection of all *ij*- δ - β -closed sets containing *S* is called the *ij*- δ - β -closure of

S and is denoted by $ij-\beta Cl^\delta(S)$. The $ij-\delta-\beta$ -interior of S is defined by the union of all $ij-\delta-\beta$ -open sets contained in S and is denoted by $ij-\beta Int^\delta(S)$. The set of all $ij-\delta-\beta$ -open sets of (X, τ^1, τ^2) is denoted by $ij-\delta\beta O(X)$. The set of all $ij-\delta-\beta$ -open sets of (X, τ^1, τ^2) containing a point $x \in X$ is denoted by $ij-\delta\beta O(X, x)$. A subset B^x of a bitopological space (X, τ^1, τ^2) is said to be an $ij-\delta-\beta$ -neighborhood of a point $x \in X$ if there exists an $ij-\delta-\beta$ -open set U such that $x \in U \subset B^x$.

Definition 2.1. [2] A subset S of a bitopological space X is called pairwise open (for short P -open) if $S \in \tau^1 \cap \tau^2$. The complement of a pairwise open set is called pairwise closed (for short P -closed). The family of all pairwise open (resp. pairwise closed) sets of X will be denoted by $Popen(X)$ (resp. $Pclosed(X)$).

Definition 2.2 [2] The union of all P -open sets of a bitopological space X contained in a subset S of X is called the P -interior of S and is denoted by $P-int(S)$. The intersection of all P -closed sets containing S is called the P -closure of S and is denoted by $P-cl(S)$.

3. PJ -SLIGHTLY $\delta-\beta$ -CONTINUOUS FUNCTION

Definition 3.1. A subset S of a bitopological space X is called pairwise clopen (for short P -clopen) if S are clopen in τ^1 and clopen in τ^2 . The complement of a pairwise clopen set is called pairwise clopen. The family of all pairwise clopen sets of X will be denoted by $Pclopen(X)$.

Definition 3.2. A subset A of X is said to be $i-\delta^*$ -open (resp. $p-\delta^*$ -open) if for each $x \in A$, there exists an i -clopen (resp. p -clopen) set U containing x such that $U \in A$. The complement of a $i-\delta^*$ -open (resp. $p-\delta^*$ -open) set is called a $i-\delta^*$ -closed (resp. $p-\delta^*$ -closed) set. The intersection of all $i-\delta^*$ -closed (resp. $p-\delta^*$ -closed) set containing A is called $i-\delta^*$ -closure (resp. $p-\delta^*$ -closure) of A and is denoted by $i-Cl^{\delta^*}(A)$ (resp. $p-Cl^{\delta^*}(A)$). The union of all $i-\delta^*$ -open (resp. $p-\delta^*$ -open) sets contained in A is called $i-\delta^*$ -interior (resp. $p-\delta^*$ -interior) of A and is denoted by $i-int^{\delta^*}(A)$ (resp. $p-int^{\delta^*}(A)$).

Definition 3.3. A function $f: X \rightarrow Y$ is said to be:

(a) pj -slightly continuous at a point $x \in X$ if for each p -clopen set V in Y containing $f(x)$, there exists a j -open set U in X containing x such that $f(U) \subset V$;

(b) pj -slightly continuous if it has the property at each $x \in X$.

Definition 3.4. A function $f: X \rightarrow Y$ is said to be:

(a) pj -slightly $\delta-\beta$ -continuous at $x \in X$ if for each p -clopen set V of Y containing $f(x)$ there exists $U \in ij-\delta\beta O(X, x)$ such that $f(U) \subset V$;

(b) pj -slightly $\delta-\beta$ -continuous if it has the property at each $x \in X$.

Theorem 3.1 If a function $f: X \rightarrow Y$ is pj -quasi $\delta-\beta$ -continuous, then it is pj -slightly $\delta-\beta$ -continuous function.

Proof. Let $x \in X$ and let V be a p -clopen set in Y containing $f(x)$. Since, f is pj -quasi $\delta-\beta$ -continuous, therefore,

there exists $U \in ij\text{-}\delta\beta O(X, x)$ such that $f(U) \subset j\text{-}Cl(V) = V$. Hence, f is pj -slightly δ - β -continuous.

Remark 3.1. The converse of the above theorem is not true in general as can be seen from the following example.

Example 3.1. Let $X = Y = \{a, b, c\}$, $\tau^1 = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$, $\tau^2 = \{X, \emptyset, \{c\}, \{a, b\}\}$, $\sigma^1 = \{X, \emptyset, \{c\}\}$ and $\sigma^2 = \{X, \emptyset, \{c\}, \{b, c\}\}$. Then the identity function $I : (X, \tau^1, \tau^2) \rightarrow (X, \sigma^1, \sigma^2)$ is $p2$ -slightly δ - β -continuous but not $p2$ -quasi δ - β -continuous.

Definition 3.4. A bitopological space X is said to be p -0-dimensional if each point of X has a p -neighbourhood basis consisting of p -clopen sets.

Theorem 3.2. If $f : X \rightarrow Y$ is pj -slightly δ - β -continuous and Y is a p -0-dimensional space, then f is pj - δ - β -continuous.

Proof. Let $x \in X$ and let V be a p -open set in Y containing $f(x)$. Since, Y is p -0-dimensional, therefore, there exists a p -clopen set W in Y such that $f(x) \in W \subset V$. Now, pj -slightly δ - β -continuity of f implies that there exists $U \in ij\text{-}\delta\beta O(X, x)$ such that $f(U) \subset W \subset V$. Hence, f is pj - δ - β -continuous.

Lemma 3.1.[1] For any bitopological space (X, τ^1, τ^2) , We have $pj\text{-}Cl^\theta(A) = p\text{-}Cl(A)$ for every j -open set A

Lemma 3.2. For any bitopological space (X, τ^1, τ^2) , every p -clopen set is pj - θ -open set.

Proof. Let A is p -clopen set then A is p -open hence A is j -open. By above lemma $pj\text{-}Cl^\theta(A) = p\text{-}Cl(A)$, so $pj\text{-}int^\theta(A) = p\text{-}int(A) = A$. Therefore A is pj - θ -open.

Theorem 3.3. If $f : X \rightarrow Y$ is pj -faintly δ - β -continuous, then f is pj -slightly δ - β -continuous.

Proof. The result is obvious from the fact that every p -clopen set is pj - θ -open

Remark 3.2. The converse of the above theorem is not, however, true in general as given by the following example.

Example 3.2. Let $X = Y = \{a, b, c\}$, $\tau^1 = \{X, \emptyset, \{c\}, \{b, c\}\}$, $\tau^2 = \{X, \emptyset, \{b\}\}$, $\sigma^1 = \{X, \emptyset, \{a\}\}$ and $\sigma^2 = \{X, \emptyset, \{a\}, \{b, c\}\}$. Then $f : X \rightarrow Y$ is pj -slightly δ - β -continuous but not pj -faintly δ - β -continuous.

Theorem 3.4. Let (X, τ^1, τ^2) and (Y, σ^1, σ^2) be bitopological spaces. The following statements are equivalent for a function $f : X \rightarrow Y$

- (a) f is pj -slightly δ - β -continuous,
- (b) $f^{-1}(V) \in ij\text{-}\delta\beta O(X)$ for each p -clopen set V in Y ,
- (c) $f^{-1}(V) \in ij\text{-}\delta\beta C(X)$ for each p -clopen set V in Y ,
- (d) $f^{-1}(V)$ is ij - δ - β -clopen in X for each p -clopen set V in Y ,

(e) $f^{-1}(V) \in ij\text{-}\delta\beta O(X)$ for each $p\text{-}\delta^*$ -open set V in Y ,

(f) $f^{-1}(V) \in ij\text{-}\delta\beta C(X)$ for each $p\text{-}\delta^*$ -closed set V in Y ,

(g) $f(ij\text{-}\beta\text{-}Cl^\delta(A)) \subset p\text{-}Cl^{\delta^*}(f(A))$ for each $A \subset X$,

(h) $ij\text{-}\beta\text{-}Cl^\delta(f^{-1}(B)) \subset f^{-1}(p\text{-}Cl^{\delta^*}(B))$ for each $B \subset Y$.

Proof. (a) $\Rightarrow$ (b): Let V be a p -clopen set in Y such that $x \in f^{-1}(V)$. Thus $f(x) \in V$. Therefore, there exists $U \in ij\text{-}\delta\beta O(X, x)$ such that $f(U) \subset V$. This implies $U \subset f^{-1}(V)$. Thus $f^{-1}(V) \in ij\text{-}\delta\beta O(X)$.

(b) $\Rightarrow$ (c): Let V be a p -clopen set in Y . then $Y \setminus V$ is also a p -clopen set in Y . This implies $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V) \in ij\text{-}\delta\beta O(X)$. Hence, $f^{-1}(V) \in ij\text{-}\delta\beta C(X)$.

(c) $\Rightarrow$ (d): Obvious.

(d) $\Rightarrow$ (a): Obvious.

(a) $\Rightarrow$ (e): Let V be a $p\text{-}\delta^*$ -open set in Y and let $x \in f^{-1}(V)$. Thus $f(x) \in V$. Therefore, there exists a p -clopen set W in Y such that $f(x) \in W \subset V$. pj -slightly δ - β -continuity of f implies that there exists $U \in ij\text{-}\delta\beta O(X, x)$ such that $f(U) \subset W \subset V$. Thus $U \subset f^{-1}(V)$. Hence, $f^{-1}(V) \in ij\text{-}\delta\beta O(X)$.

(e) $\Rightarrow$ (f): Let V be a $p\text{-}\delta^*$ -closed set in Y . Thus $Y \setminus V$ is $p\text{-}\delta^*$ -open in Y . This implies that $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V) \in ij\text{-}\delta\beta O(X)$. Hence, $f^{-1}(V) \in ij\text{-}\delta\beta C(X)$.

(f) $\Rightarrow$ (g): $p\text{-}Cl^{\delta^*}(f(A))$ is a $p\text{-}\delta^*$ -closed set in Y containing $f(A)$. Thus $f^{-1}(p\text{-}Cl^{\delta^*}(f(A)))$ is a $ij\text{-}\delta\text{-}\beta$ -closed set in X containing A . Thus $ij\text{-}\beta\text{-}Cl^\delta(A) \subset f^{-1}(p\text{-}Cl^{\delta^*}(f(A)))$. Hence $f(ij\text{-}\beta\text{-}Cl^\delta(A)) \subset p\text{-}Cl^{\delta^*}(f(A))$.

(g) $\Rightarrow$ (h): Let $B \subset Y$, then $f^{-1}(B) \subset X$. Thus $f(ij\text{-}\beta\text{-}Cl^\delta(f^{-1}(B))) \subset p\text{-}Cl^{\delta^*}(f(f^{-1}(B))) \subset p\text{-}Cl^{\delta^*}(B)$. Hence, $ij\text{-}\beta\text{-}Cl^\delta(f^{-1}(B)) \subset f^{-1}(p\text{-}Cl^{\delta^*}(B))$.

(h) $\Rightarrow$ (c): Let V be a p -clopen set in Y . Thus V is $p\text{-}\delta^*$ -closed in Y . Therefore $ij\text{-}\beta\text{-}Cl^\delta(f^{-1}(V)) \subset f^{-1}(p\text{-}Cl^{\delta^*}(V)) = f^{-1}(V)$. Hence, $f^{-1}(V) \in ij\text{-}\delta\beta C(X)$.

Theorem 3.5. If $f : X \rightarrow Y$ is pj -slightly δ - β -continuous and U is pj - δ -open, then $f|_U : U \rightarrow Y$ is pj -slightly δ - β -continuous.

Proof. Let V be p -clopen set. Since, f is pj -slightly δ - β -continuous, therefore, $f^{-1}(V) \in ij\text{-}\delta\beta O(X)$. Now, $(f|_U)^{-1}(V) = f^{-1}(V) \cap U \in ij\text{-}\delta\beta O(U)$.

$f^{-1}(V) = f^{-1}(V) \cap U \in ij\text{-}\delta\beta O(U)$, (by lemma 4.1). Therefore, $f|_U$ is pj -slightly δ - β -continuous.

Lemma 3.3. Let $A \subset U \subset X$, $A \in ij\text{-}\delta\beta O(U)$ and U is i -open in X , then $A \in ij\text{-}\delta\beta O(X)$.

Proof. We have

$$A \subset j\text{-}Cl^U(i\text{-}int^U(ij\text{-}Cl^{\partial U}(A))) \subset j\text{-}Cl(i\text{-}int^U(ij\text{-}Cl^{\partial U}(A))) = j\text{-}Cl(i\text{-}int(ij\text{-}Cl^{\partial U}(A)) \cap U) = j\text{-}Cl(i\text{-}int(ij\text{-}Cl^{\partial U}(A)) \cap i\text{-}int(U)) = j\text{-}Cl(i\text{-}int(ij\text{-}Cl^{\partial U}(A))) \subset j\text{-}Cl(i\text{-}int(ij\text{-}Cl^{\delta}(A))). \text{ Hence, } A \in ij\text{-}\delta\beta O(X).$$

Theorem 3.6. Let $f: X \rightarrow Y$ be a function and let $\{U^\alpha : \alpha \in A\}$ be an i -open cover of X . If $f|_{U^\alpha}$ is pj -slightly δ - β -continuous for each $\alpha \in A$, then f is a pj -slightly δ - β -continuous function.

Proof. Suppose that V is any p -clopen set in Y . Since $f|_{U^\alpha}$ is pj -slightly δ - β - continuous for each $\alpha \in A$, it follows that $(f|_{U^\alpha})^{-1}(V) \in ij\text{-}\delta\beta O(U^\alpha)$. We have, $f^{-1}(V) = \bigcup_{\alpha \in A} (f|_{U^\alpha})^{-1}(V) \cap U^\alpha = \bigcup_{\alpha \in A} f|_{U^\alpha}^{-1}(V)$. Then, by Lemma 5.2 we obtain $f^{-1}(V) \in ij\text{-}\delta\beta O(X)$. Hence, f is pj -slightly δ - β -continuous.

Theorem 3.7. Let $f: X \rightarrow Y$ be a function and let $x \in X$. If there exists an i -open set U in X such that $x \in U$ and $f|_U$ is a pj -slightly δ - β -continuous function at x , then f is pj -slightly δ - β -continuous at x .

Proof. Suppose that V be a p -clopen set in Y containing $f(x)$. Since $f|_U$ is pj -slightly δ - β -continuous at x , there exists $W \in ij\text{-}\delta\beta O(U, x)$ such that $f(W) = (f|_U(W)) \subset V$. Since, U is i -open in X containing x , it follows from Lemma 5.2 that $W \in ij\text{-}\delta\beta O(X, x)$. This shows that f is pj -slightly δ - β -continuous at x .

Theorem 3.8. Let $f: X \rightarrow Y$ be a function and let $g: X \rightarrow X \times Y$ be the graph function of f , defined by $g(x) = (x, f(x))$ for every $x \in X$. Then g is pj -slightly slightly δ - β -continuous if and only if f is pj -slightly δ - β -continuous

Proof. Let g be pj -slightly δ - β -continuous and let V be a p -clopen set in Y . Then $X \times V$ is p -clopen in $X \times Y$. Since g is pj -slightly δ - β -continuous, thus $f^{-1}(V) = g^{-1}(X \times V) \in ij\text{-}\delta\beta O(X)$. Thus, f is pj -slightly δ - β -continuous.

Conversely, let $x \in X$ and let W be a p -closed subset of $X \times Y$ containing $g(x)$. Then $W \cap (\{x\} \times Y)$ is p -clopen in $\{x\} \times Y$ containing $g(x)$. Also $\{x\} \times Y$ homeomorphic to Y . Hence $\{y \in Y : (x, y) \in W\}$ is a p -clopen subset of Y . Since f is pj -slightly δ - β -continuous, $\bigcup \{f^{-1}(y) : (x, y) \in W\}$ is an $ij\text{-}\delta$ - β -open subset of X . Further $x \in \bigcup \{f^{-1}(y) : (x, y) \in W\} \subset g^{-1}(W)$. Then $g^{-1}(W)$ is $ij\text{-}\delta$ - β -open. Hence g is pj -slightly δ - β -continuous.

Definition 3.5 A function $f: X \rightarrow Y$ is called $ij\text{-}\delta$ - β -irresolute if for every $ij\text{-}\delta$ - β -open set G in Y , $f^{-1}(G)$ is an $ij\text{-}\delta$ - β -open set in X .

Definition 3.6 A function $f: X \rightarrow Y$ is called $ij\text{-}\delta$ - β -open if for every $ij\text{-}\delta$ - β -open set H in X , $f(H)$ is a $ij\text{-}\delta$ - β -open set in Y .

Remark 3.3. Composition of two pj -slightly $\delta - \beta$ -continuous functions may not be pj -slightly $\delta - \beta$ -continuous as given by the following example.

Example 3.3. Let $X = Y = Z = \{a, b, c\}$ and let

$$\tau^1 = \{X, \emptyset, \{b\}\}, \tau^2 = \{X, \emptyset, \{c\}, \{a, b\}\},$$

$$\sigma^1 = \{Y, \emptyset, \{a\}, \{c\}, \{a, b\}\}, \sigma^2 = \{Y, \emptyset, \{c\}, \{b, c\}, \{a\}\},$$

$$\gamma^1 = \{Z, \emptyset, \{a\}, \{b, c\}, \{c\}\}, \gamma^2 = \{Z, \emptyset, \{a\}, \{b, c\}\}.$$

Then the functions $f: (X, \tau^1, \tau^2) \rightarrow (Y, \sigma^1, \sigma^2)$ and $g: (Y, \sigma^1, \sigma^2) \rightarrow (Z, \gamma^1, \gamma^2)$ defined by identity function are both $p2$ - $\delta - \beta$ -continuous but $g \circ f: (X, \tau^1, \tau^2) \rightarrow (Z, \gamma^1, \gamma^2)$ is not $p2$ -slightly $\delta - \beta$ -continuous.

Definition 3.7. A function $f: X \rightarrow Y$ is said to be:

(a) jp -slightly $\delta - \beta$ -continuous at $x \in X$ if for each p -clopen set V of Y containing $f(x)$ there exists $U \in ji\text{-}\delta\beta O(X, x)$ such that $f(U) \subset V$;

(b) jp -slightly $\delta - \beta$ -continuous if it has the property at each $x \in X$.

Theorem 3.9. Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be functions. Then, the following properties hold:

(a) If f is ij - $\delta - \beta$ -irresolute and g is pj -slightly $\delta - \beta$ -continuous, then $g \circ f$ is pj -slightly $\delta - \beta$ -continuous.

(b) If f is ij - $\delta - \beta$ -irresolute and g is pj - $\delta - \beta$ -continuous, then $g \circ f$ is pj -slightly $\delta - \beta$ -continuous.

(c) If f is ji - $\delta - \beta$ -irresolute and g is pj -slightly-continuous, then $g \circ f$ is jp -slightly $\delta - \beta$ -continuous.

Proof. (a) Let V be any p -clopen set in Z . Since g is pj -slightly $\delta - \beta$ -continuous, $g^{-1}(V)$ is ij - $\delta - \beta$ -open. Since, f is ij - $\delta - \beta$ -irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is ij - $\delta - \beta$ -open. Therefore, $g \circ f$ is pj -slightly $\delta - \beta$ -continuous.

(b) It follows from the fact that every p -clopen set is p -open.

(c) It follows from the fact that every j -open set is ji - $\delta - \beta$ -open.

Theorem 3.10. If $f: X \rightarrow Y$ is ij - $\delta - \beta$ -open and surjective and $g \circ f: X \rightarrow Z$ is pj -slightly $\delta - \beta$ -continuous, then $g: Y \rightarrow Z$ is pj -slightly $\delta - \beta$ -continuous

Proof. Let V be any p -clopen set in Z . Since $g \circ f$ is pj -slightly $\delta - \beta$ -continuous, therefore, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ij - $\delta - \beta$ -open in X . Since, f is ij - $\delta - \beta$ -open and surjective, therefore, $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is ij - $\delta - \beta$ -open. Hence, g is pj -slightly $\delta - \beta$ -continuous

Remark 3.4. The condition surjectiveness cannot be dropped from f as shown by the following example:

Example 3.4. Let $X = Y = Z = \{a, b, c\}$ and let

$$\tau^1 = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}, \tau^2 = \{X, \emptyset, \{a, b\}\},$$

$$\sigma^1 = \{Y, \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}, \sigma^2 = \{Y, \emptyset, \{a\}, \{c\}, \{a, c\}\},$$

$$\gamma^1 = \{Z, \emptyset, \{b\}, \{a, c\}\}, \gamma^2 = \{Z, \emptyset, \{a\}, \{b\}, \{a, c\}, \{a, b\}\}.$$

Let the function $f: (X, \tau^1, \tau^2) \rightarrow (Y, \sigma^1, \sigma^2)$ and $g: (Y, \sigma^1, \sigma^2) \rightarrow (Z, \gamma^1, \gamma^2)$ be defined by $f(a) = f(c) = a$, $f(b) = c$ and $g(x) = x$ for all $x \in Y$ respectively. Then f is 12 - δ - β -open, $g \circ f$ is $p2$ -slightly δ - β -continuous but g is not $p2$ -slightly δ - β -continuous at $b \in Y$.

Theorem 3.11. Let $f: X \rightarrow Y$ be surjective, ij - δ - β -irresolute and ij - δ - β -open and $g: Y \rightarrow Z$ be a function. Then $g \circ f$ is pj -slightly δ - β -continuous if and only if g is pj -slightly δ - β -continuous.

Definition 3.8. A bitopological space X is called p -connected if X cannot be expressed as the union of two disjoint non-empty p -open sets.

Theorem 3.12. If $f: X \rightarrow Y$ is pj -slightly δ - β -continuous surjection and X is ij - δ - β -connected space, then Y is p -connected space.

Proof. Suppose that Y is not a p -connected space. Then there exists non-empty disjoint p -open sets U and V such that $Y = U \cup V$. Therefore, U and V are p -clopen sets in Y . Since f is pj -slightly δ - β -continuous, then $f^{-1}(U), f^{-1}(V)$ are ij - δ - β -open in X . Moreover, $f^{-1}(U)$ and $f^{-1}(V)$ are non-empty disjoint and $X = f^{-1}(U) \cup f^{-1}(V)$. This shows that X is not ij - δ - β -connected. This is a contradiction. Hence, Y is p -connected.

Definition 3.9. A bitopological space X is said to be:

(a) p -clopen T^1 if for each pair of distinct points x and y of X , there exist p -clopen sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.

(b) p -clopen T^2 (p -clopen Hausdorff) if for each pair of distinct points x and y of X , there exist disjoint p -clopen sets U and V such that $x \in U$ and $y \in V$.

Theorem 3.13. If $f: X \rightarrow Y$ is a pj -slightly δ - β -continuous injection and Y is p -clopen T^1 , then X is ij - δ - β - T^1 .

Proof. Let Y be p -clopen T^1 and let x and y be two distinct points of X . Since, f is an injection, therefore, $f(x) \neq f(y)$ in Y . Also, p -clopen T^1 -ness of Y implies that there exist p -clopen sets V and W in Y such that $f(x) \in V$, $f(y) \notin V$, $f(x) \notin W$ and $f(y) \in W$. Since, f is pj -slightly δ - β -continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are ij - δ - β -open subsets of X such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $x \notin f^{-1}(W)$ and $y \in f^{-1}(W)$. This shows that X is ij - δ - β - T^1 .

Theorem 3.14. If $f: X \rightarrow Y$ is a pj -slightly δ - β -continuous injection and Y is p -clopen T^2 , then X is ij - δ - β - T^2 .

Proof. Let Y be a p -clopen T^2 space and let x, y be two distinct points of X . Since, f is an injection, therefore, $f(x) \neq f(y)$ in Y . Therefore, there exist disjoint p -clopen sets V and W in Y such that $f(x) \in V$ and $f(y) \in W$. Since, f is pj -slightly δ - β -continuous, therefore, $f^{-1}(V) \in ij\text{-}\delta\beta O(X, x)$ and $f^{-1}(W) \in ij\text{-}\delta\beta O(X, y)$ and $f^{-1}(V) \cap f^{-1}(W) = \emptyset$. Hence, X is $ij\text{-}\delta - \beta - T^2$.

Definition 3.10. A bitopological space X is said to be pj -clopen regular ($ij\text{-}\delta - \beta$ -regular) if for each p -clopen (resp. $ij\text{-}\delta - \beta$ -closed) set F and each point $x \notin F$, there exist i -open sets U and j -open V such that $x \in U$ and $F \subset V$.

Definition 3.11. A bitopological space X is said to be pj -clopen normal ($ij\text{-}\delta\text{-}\beta$ -normal) if for every pair of disjoint p -clopen (resp. $ij\text{-}\delta\text{-}\beta$ -closed) sets F^1, F^2 , there exist i -open set U and j -open set V such that $F^1 \subset U$ and $F^2 \subset V$.

Theorem 3.15. If f is pj -slightly $\delta - \beta$ -continuous injective open function from an $ij\text{-}\delta - \beta$ -regular space X onto a space Y , then Y is pj -clopen regular.

Proof. Let F be a p -clopen set in Y and let $y \notin F$. Since, f is onto, there exists $x \in X$ such that $f(x) = y$. Since f is pj -slightly $\delta - \beta$ -continuous, $f^{-1}(F)$ is an $ij\text{-}\delta - \beta$ -closed set. We have, $x \notin f^{-1}(F)$. Since, X is $ij\text{-}\delta\text{-}\beta$ -regular, there exist j -open sets U and i -open set V such that $f^{-1}(F) \subset U$ and $x \in V$. We obtain that $F = f(f^{-1}(F)) \subset f(U)$ and $y = f(x) \in f(V)$ such that $f(U)$ is j -open set and $f(V)$ is i -open set. This shows that Y is pj -clopen regular

Theorem 3.16. If f is pj -slightly $\delta - \beta$ -continuous injective open function from an $ij\text{-}\delta - \beta$ -normal space X onto a space Y , then Y is pj -clopen normal.

Proof. Let F^1 and F^2 be two disjoint p -clopen subsets of Y . Since f is pj -slightly $\delta - \beta$ -continuous, $f^{-1}(F^1)$ and $f^{-1}(F^2)$ are disjoint $ij\text{-}\delta - \beta$ -closed sets. Since, X is $ij\text{-}\delta - \beta$ -normal, there exist i -open set U and j -open set V such that $f^{-1}(F^1) \subset U$ and $f^{-1}(F^2) \subset V$. Thus, we obtain that $F^1 = f(f^{-1}(F^1)) \subset f(U)$ and $F^2 = f(f^{-1}(F^2)) \subset f(V)$ such that $f(U)$ is i -open set and $f(V)$ is j -open set. Thus, Y is pj -clopen normal.

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